for a general conservation variable F, and thus the appropriate finite difference analog is

$$\sum_{n=0}^{N} \frac{\partial}{\partial \psi}\left(a\frac{\partial F}{\partial \psi}\right)\Delta\psi = 0 \tag{8}$$

and not Eq. (5) as used by Sukanek.[2]

References

[1]Sukanek, P.C., and Rhodes, R.P., "Centerline Formulation in the Numerical Computation of Axisymmetric Flows," *AIAA Journal,* Vol. 16, Oct. 1978, pp. 1099-1101.

[2]Sukanek, P.C., "Conservation Errors in Axisymmetric Finite-Difference Equations," *AIAA Journal,* Vol. 17, Jan. 1979, pp. 99-101.

Reply by Author to R. Edelman and P.T. Harsha

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EDELMAN and Harsha present two objections to our work. They claim, first, that the centerline formulation reported in Ref. 1 is not second-order accurate and introduces a "spurious" diffusion term; second, that the conservation statement employed in Ref. 2 is incorrect. These objections are interrelated, since it is our contention that the "spurious" diffusion term is necessary to preserve conservation.

The finite differrence analog of the global conservation property used in Ref. 2 is identical to that reported by Edelman and Harsha, provided a consistent choice of ψ_n is employed:

$$\sum_{n=0}^{N}\left\{\frac{1}{\psi}\frac{\partial}{\partial\psi}\left(a\frac{\partial F}{\partial\psi}\right)\right\}_n \psi_n\Delta\psi \equiv \sum_{n=0}^{N}\left\{\frac{\partial}{\partial\psi}\left(a\frac{\partial F}{\partial\psi}\right)\right\}_n\Delta\psi \tag{1}$$

The notation $\{\ \}_n$ refers to the finite difference form of the quantity within the braces. The left side of this identity was used in Ref. 2 to facilitate the examination of conservation when a limiting form of the equation is used at the axis of symmetry. In this case, the quantity on the left side in braces must be replaced by its limit as ψ goes to zero.

The right or left side of Eq. (1) may be used to obtain an expression for the centerline which guarantees overall conservation. Both give the same result, Eq. (13) of Ref. 2, indicating that the appropriate value of the parameter a, the diffusion coefficient, is the average of a between the centerline and the first radial position. This is in agreement with the centerline formulation derived in Ref. 1 using an integral approach. The same formulation can be found using the control volume method, i.e., expressing the balance among convection, diffusion, and production of the quantity F for the center streamtube. The diffusion term must be evaluated at the streamtube boundary. Finally, the result also agrees with the intuitive concept that the flux of material leaving tube n must equal the flux entering tube $(n+1)$. A limiting form of the conservation equation, where the diffusion coefficient is evaluated at the centerline and not the streamtube boundary, does not permit this equality.

Edelman and Harsha use the limiting form of the governing equations at the centerline. This form is exact only for each point on the axis of symmetry. However, the finite difference form of the equation must be valid not for a point, but for some finite region surrounding the point. In the appropriate limits, Eq. (7) of Ref. 1 does reduce to the correct equation:

$$\lim_{\substack{\Delta X\to 0\\ \Delta\psi\to 0}}\left[\frac{F_{n+1,0}-F_{n,0}}{\Delta X} - \frac{4a_+}{b\Delta\psi}\frac{2(F_{n,1}-F_{n,0})}{(\Delta\psi)^2} - P_{n,0}\right]$$

$$= \frac{\partial F}{\partial X} - \frac{2\mu_o}{b}\frac{\partial^2 F}{\partial\psi^2} - P \tag{2}$$

While the limiting form of the conservation equations at the centerline is exact, this alone does not guarantee that the corresponding finite difference equations will also be exact.

References

[1]Sukanek, P.C. and Rhodes, R.P., "Centerline Formulation in the Numerical Computation of Axisymmetric Flows," *AIAA Journal,* Vol. 16, Oct. 1978, pp. 1099-1101.

[2]Sukanek, P.C., "Conservation Errors in Axisymmetric Finite-Difference Equations," *AIAA Journal,* Vol. 17, Jan. 1979, pp. 99-101.

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